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(1)

Exercice n° 3) : 3) :

$$R\cos(w.t)\,\vec{u_x} + R\sin(w.t)\,\vec{u_y} = 0.$$

1) $\vec{v} = -Rw\sin(w.t)\,\vec{v_x} + Rw\cos(wt)\,\vec{v_y}$

$\vec{a} = -Rw^2\cos(w.t)\,\vec{a_x} - Rw^2\sin(w.t)\,\vec{a_y}$

3)

$\vec{a}.\vec{v} = a_x v_x + a_y v_y$

$\vec{a}.\vec{v} = (-Rw^2\cos(w.t))(-Rw\sin(w.t)) + (-Rw^2\sin(wt))(Rw\cos(wt))$

$\vec{a}.\vec{v} = R^2w^3\cos(w.t)\sin(wt) - R^2w^3\sin(wt)\cos(wt)$

$\vec{a}.\vec{v} = 0 \quad \forall t$

$\vec{a} \perp \vec{v} \quad \forall t$

4) $\begin{cases} x(t) = R\cos(w.t) \\ y(t) = R\sin(wt) \end{cases} \Rightarrow \begin{cases} x^2 = R^2\cos^2(wt) \\ y^2 = R^2\sin^2(w.t) \end{cases}$

$\Rightarrow x^2 + y^2 = R^2\cos^2(w.t) + R^2\sin^2(wt) = R^2\underbrace{(\cos^2(w.t) + \sin^2(wt))}_{1\ \forall t}$

$x^2 + y^2 = R^2$

$\underbrace{(x - x_0)^2 + (y + y_0)^2}_{\text{cercle de centre } (x_0 ; y_0)} = R^2$ cercle de rayon R de centre O.

5) Cartésiens $\rightarrow$ polaire

$\begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} \longrightarrow \rho(t)\sqrt{(x(t))^2 + y(t)^2}$

$OM(t) = \rho(t)\,\vec{u_\rho}$

$\rho = \sqrt{\underbrace{R^2\cos^2(w.t)}_{x(t)^2} + \underbrace{R^2\sin^2(wt)}_{y(t)^2}}$ (Pythagore)

$\rho = R\underbrace{\sqrt{\cos^2(wt) + \sin^2(wt)}}_{=1} = R = cste$

$\boxed{\vec{OM} = R\vec{u_\rho}} \rightarrow \vec{V} = \frac{d\vec{OM}}{dt} = \frac{d}{dt}(R\vec{u_\rho})$

$= R\frac{d\vec{u_\rho}}{dt} = R\dot{\theta}\vec{u_\theta}$

$\vec{V} = R\omega\,\vec{u_\theta}$.

$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\underbrace{R\omega}_{\theta}\vec{u_\theta}) = R\omega\,\frac{d\vec{u_\theta}}{dt}$ car $R\omega = cst$

$\vec{a} = -R\omega^2\vec{u_\rho}$

$\hookrightarrow -\dot{\theta}\vec{u_\rho} = -\omega\vec{u_\rho}$

Exercice 4 :

Partie A :

$\vec{OM}\begin{cases} x(t) = A\cos(\omega.t) \\ y(t) = B\sin(\omega.t) \\ z(t) = (H\omega.t) \end{cases}$ A, B, H, ω cst > 0

1) $\vec{V}\begin{cases} V_x = \dot{x} = -A\omega\sin(\omega t) \\ V_y = \dot{y} = B\omega\cos(\omega t) \\ V_z = \dot{z} = H\omega \end{cases}$

2) $\vec{a}\begin{cases} a_x = \dot{V_x} = -A\omega^2\cos(\omega.t) \\ a_y = \dot{V_y} = -B\omega^2\sin(\omega t) \\ a_z = 0 \end{cases}$

3) Normes:

$\vec{v} = \sqrt{V_x^2 + V_y^2 + V_z^2}$

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4a) $\begin{cases} x(t) = A\cos(w.t) \Rightarrow \cos(w.t) = \frac{x}{A} \Rightarrow \cos^2(wt) = \frac{x^2}{A^2} \\ y(t) = B\sin(wt) \Rightarrow \sin(w.t)\ \frac{y}{B} \Rightarrow \cos^2(wt) = \frac{y^2}{B^2} \end{cases}$

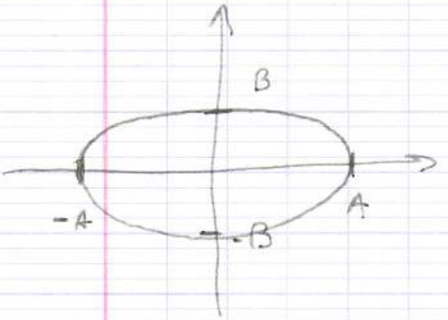
$$\Rightarrow \frac{x^2}{A^2} + \frac{x^2}{B^2} = \cos^2(w.t) + \sin^2(w.t) = 1$$

$\left| \frac{x^2}{A^2} + \frac{x^2}{B^2} = 1 \right)$ ellipse (traj. elliptique dans le plan $(x\,oy)$)

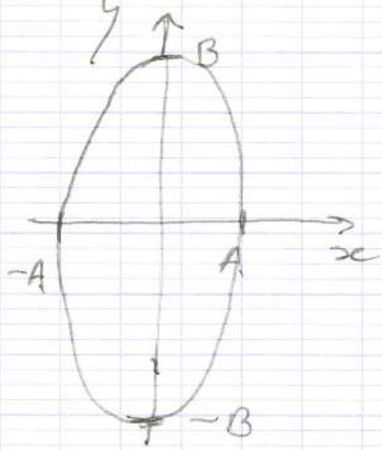
Si $A > B$:

Si $x=0$ $y = \pm B$

Si $y=0$ $x \pm A$



Si $A < B$:



b) Sur l'axe Oz

$\begin{cases} V_z = Hw = \text{cste} \\ z = (Hw)t = V.t \end{cases}$ $\Rightarrow$ Mt rectiligne uniforme sur l'axe Oz

c) le mouvement resultant est un mvt hélicoïdal elliptique

PARTIE B =

$\begin{cases} A = B = R \\ \text{base cylindrique} \neq \text{galiléenne} \end{cases} \Rightarrow \begin{cases} x(t) = R\cos(wt) \\ y(t) = R\sin(wt) \\ z(t) = (Hw)t \end{cases} \Rightarrow \begin{cases} \rho(t) = \sqrt{R^2\cos^2(wt) + R^2\sin^2(wt)} = R \\ z(t) = (Hw)t \end{cases}$

$$\overrightarrow{OM}_{cyl} = \rho \, \vec{u}_\rho + z \, \vec{u}_z$$

$$\overrightarrow{OM}_{cyl} = R \, \vec{u}_\rho + (H\omega) t \, \vec{u}_z$$ (hélicoïdale circulaire)

$$\vec{v} = \frac{d\overrightarrow{OM}}{dt} = \frac{d}{dt}\left(R\vec{u}_\rho + H\omega t \, \vec{u}_z\right) = R \underbrace{\frac{\vec{u}_\rho}{dt}}_{\dot\theta \vec{u}_\theta} + \theta H\omega \, \vec{u}_z$$

$$\vec{v} = \overbrace{(R\omega)}^{v_\theta} \, \vec{u}_\theta + \overbrace{(H\omega)}^{v_z} \, \vec{u}_z$$

$\dot\theta = \omega$ car $\dot\theta$ cste

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(R\omega \, \vec{u}_\theta + H\omega \, \vec{u}_z\right)$$

$$= R\omega \, \dot{\vec{u}}_\theta = R\omega \left(- \underset{\omega}{\underset{\shortparallel}{\dot\theta}} \, \vec{u}_\rho\right) = - R\omega^2 \, \vec{u}_\rho .$$