Physique

16/10

abscisse curviligne élementaire

$$ds = R\,d\theta$$

$$\vec{v} = \frac{ds}{dt}\,\vec{u_T} = \frac{R\,d\theta}{dt}\,\vec{u_T} = R \cdot \dot{\theta}\,\vec{u_T}$$

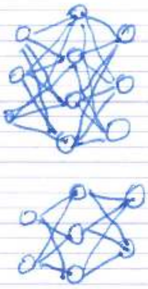
$$s(t_2) = \int_{t=0}^{t_2} ds = \int_{t=0}^{t_2} v\,dt$$

$$\vec{a} = \underbrace{\frac{d\vec{v}}{dt}}_{a_T}\,\vec{u_T} + \underbrace{\frac{v^2}{R}}_{a_N}\,\vec{u_N}$$

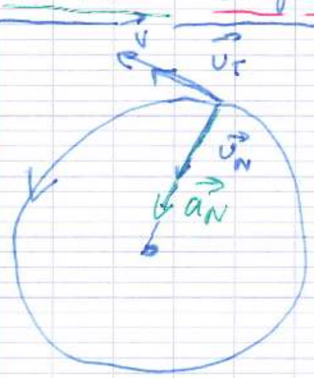
TD 3 : 

Exercice n°1 2)

$$\vec{a}\begin{pmatrix} \frac{d\vec{v}}{dt} = a_T \\ \frac{v^2}{r} = a_N \end{pmatrix}$$



Circulaire uniforme



$\Downarrow$

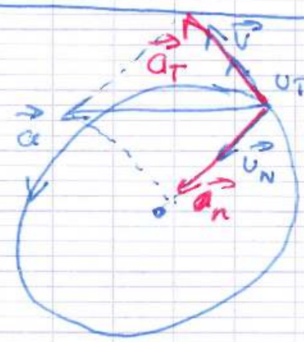
$v = cste$

$\frac{dv}{dt} = 0$

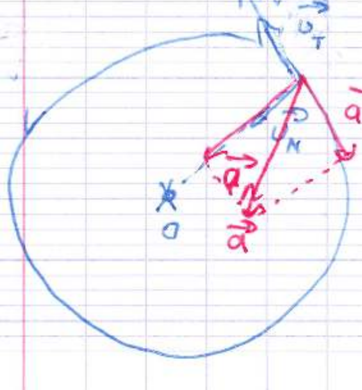
$a_T = 0$

$a = a_N$

Circulaire accelérée



Circulaire décéléré



2 8 2

Exercice n°2

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1) $\begin{cases} x(t) = 2t \\ y(t) = \sqrt{4(1-t^2)} \end{cases}$

Trajectoire

② $\Rightarrow y^2 = 4(1-t^2) = 4 - \underbrace{4t^2}_{x^2}$

① $\Rightarrow x^2 = 4t^2$

$\Rightarrow y^2 = 4 - x^2 \Leftrightarrow x^2 + y^2 = 4$

cercle de centre O et de rayon $R = \sqrt{4} = 2$

$\left((x-x_0)^2 + (y-y_0)^2 = R^2 \right)$

cas où le centre $O(x_0, y_0)$

2) $\vec{V} = \dfrac{d\vec{OM}}{dt} = \begin{cases} \dot{x} = V_x = 2 \\ \dot{y} = V_y = \dfrac{-8t}{2\sqrt{4(1-t^2)}} \end{cases}$

$= \left(\sqrt{u}\right)' = \dfrac{1}{2\sqrt{u}} \times u'$

$\vec{V} = \begin{pmatrix} 2 \\ \dfrac{-2t}{\sqrt{1-t^2}} \end{pmatrix} \begin{matrix} v_x \\ v_y \end{matrix}$

Norme de $\vec{v}$

$v = \sqrt{V_x^2 + V_y^2} = \sqrt{4 + \dfrac{4t^2}{(1-t^2)}}$

$= \sqrt{\dfrac{4 - 4t^2 + 4t^2}{1-t^2}} =$

$v(t) = \dfrac{2}{\sqrt{1-t^2}}$ (norme est la même pour toute la base)

2b) $\begin{cases} a_T = \dfrac{dv}{dt} = \dfrac{d}{dt}\left(2(1-t^2)^{-\frac{1}{2}}\right) = 2\left(-\dfrac{1}{2}\right)(-2t)(1-t^2)^{-\frac{1}{2}-1} \\ a_N = \dfrac{v^2}{r} = \dfrac{4}{1-t^2} \times \dfrac{1}{2} = \dfrac{2}{1-t^2} \end{cases}$

$\begin{cases} a_T = \dfrac{2t}{(1-t^2)^{\frac{3}{2}}} \\ a_N = \dfrac{2}{1-t^2} \end{cases}$ Composantes en Base de Frenet