Maths
17/05
1

3.5) $F = \dfrac{3}{(2X-1)(X^2+1)} = \dfrac{a}{2X-1} + \dfrac{bX+c}{X^2+1}$

$\left[(2X-1)F\right]_{X=0,5} = \dfrac{3}{1,25} = a \Rightarrow a = \dfrac{12}{5}$

Pour b et c :

1ère méthode : $\lim_{X \to +\infty} XF = 0 = \dfrac{a}{2} + b \Rightarrow b = \dfrac{-a}{2} = \dfrac{-6}{5}$

$F(0) = -3 = -a + c \Rightarrow c = -3 + a = -3 + \dfrac{12}{5} = -\dfrac{3}{5}$

2e méthode :

$(X^2+1)F = \dfrac{3}{2X-1} = \dfrac{a(X^2+1)}{2X-1} + bX + c$

$\Rightarrow \left[(X^2+1)F\right]_{X=i} = \dfrac{3}{2i-1} = bi + c$

$\Rightarrow bi + c = \dfrac{3(-2i-1)}{(2i-1)(-2i-1)} = \dfrac{-6i-3}{5} = \dfrac{-6}{5}i - \dfrac{3}{5}$

$\Rightarrow b = \dfrac{-6}{5}$ et $c = \dfrac{-3}{5}$

CCL : $F = \dfrac{\frac{12}{5}}{2X-1} + \dfrac{\frac{-6}{5}X + \frac{-3}{5}}{X^2+1}$

4)

$F = \dfrac{X^6+1}{(X-1)(X^2+1)^2}$

Partie entière :

Dénominateur : $(X^4+2X^2+1)(X-1) = (X^5+2X^3+X-X^4-2X^2-1)$

$$
\begin{array}{l|l}
X^6 \qquad\qquad\qquad\qquad\qquad +1 & X^5-X^4+2X^3-2X^2+X-1 \\
-\cancel{X^6}+X^5-2X^4+2X^3-X^2+\cancel{X} & \hline X+1 \\
\hline
X^5-2X^4+2X^3-X^2+X+1 & \\
-X^5+X^4-2X^3+2X^2-X+1 & \\
\hline
-X^4+X^2+2 &
\end{array}
$$

$$F = \overbrace{X+1}^{E(X)} + \overbrace{\frac{-X^4+X^2+2}{(X-1)(X^2+1)^2}}^{F_1}$$

$$F_1 = \frac{-X^4+X^2+2}{(X-1)(X^2+1)^2} = \frac{a}{X-1} + \frac{bX+c}{X^2+1} + \frac{dX+e}{(X^2+1)^2}$$

$$(X^2+1)^2 F_1 = \frac{(-X^4+X^2+1)\cancel{(X^2+1)^2}}{(X-1)\cancel{(X^2+1)^2}} = \frac{a(X^2+1)^2}{(X-2)} + \frac{(bX+c)(X^2+1)^{\cancel{2}}}{\cancel{(X^2+1)}} + dX+e$$

$$= \frac{-X^4+X^2+2}{X-1} = \frac{a(X^2+1)}{X-1} + (bX+c)(X^2+1) + dX+e$$

$$\left[(X-1)F_1\right]_{X=1} = \frac{-1+1+2}{2^2} = a \Rightarrow a = \frac{1}{2}$$

$$\left[(X^2+1)^2F_1\right]_{X=i} = \frac{-1-1+2}{i-1} = dX+e \Rightarrow di+e = 0 \Leftrightarrow d=e=0$$

Calcul de b et c :

$$\lim_{X\to+\infty} XF_1 = -1 = a+b$$

$$\Rightarrow b = -1-a = \frac{-3}{2}$$

$$F_1(0) = -\frac{1}{2} + \frac{c}{1} = \frac{2}{-1\times 1} = -2$$

$$\Rightarrow c = -2+\frac{1}{2} = \frac{-3}{2}$$

$$\text{CCL : } F = X+1 + \frac{\frac{1}{2}}{X-1} + \frac{\frac{-3}{2}X - \frac{3}{2}}{X^2+1}.$$

Rq la fraction se simplifiait !

$P(X) = X^6+1$

$P(i) = 0$ d'où $X-i \mid P$

$P(-i) = 0$ d'où $X+i \mid P$

$\Rightarrow (X-i)(X+i) \mid P \Rightarrow X^2+1 \mid P$ càd $P = (X^2+1)(X^4-X^2+1)$

$$F = \frac{X^4-X^2+1}{(X-1)(X^2+1)}$$