Série 13 : Thermodynamique 14/05

Exercice n°1 : 1) $PV^\gamma = \text{cste}$ (Loi de Laplace) (adiabatique).

$P = \frac{nRT}{V}$ (Loi des G.P.)

$\frac{nRT}{V} V^\gamma = \text{cste}$

$T.V^{\gamma-1} = \frac{\text{cste}}{nR} = \text{cste}.$

$T.V^{\gamma-1} = \text{cste}.$

Pour DA :

$T_A . V_A^{\gamma-1} = T_D . V_D^{\gamma-1}$ or $V_A = V_1$ et $V_D = V_2$

$\Rightarrow T_A V_1^{\gamma-1} = T_D V_2^{\gamma-1}$

De même pour le trajet adiabatique BC

$T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1}$ or $V_B = V_2$ et $V_C = V_2$

$\Rightarrow T_B V_1^{\gamma-1} = T_C V_2^{\gamma-1}$

2) 3)

Transf	$W = -\int_i^f P dV$	$Q = \Delta U - W$ (1er principe)	$\Delta U = nc_v \Delta T$
$D \to A$ adiabatique	$W_{DA} = nc_v (T_A - T_D)$ ($W = \Delta U$ car $Q = 0$)	$Q_{DA} = 0$ (adiabatique)	$\Delta U_{DA} = nc_v (T_A - T_D)$
$A \to B$ isochore	$W_{AB} = 0$	$Q_{AB}\ nc_v (T_B - T_A)$ > 0	$\Delta U_{AB} = nc_v (T_B - T_A)$
$B \to C$ adiab	$W_{BC} = nc_v (T_C - T_B)$ ($W_{BC} = \Delta U_{BC}$ car $Q = 0$)	$Q_{BC} = 0$	$\Delta U_{BC} = nc_v (T_C - T_B)$
$C \to D$ isochore	$W_{CD} = 0$	$Q_{CD} = \Delta U_{CD} < 0$ $Q_{CD} = nc_v (T_D - T_C)$	$\Delta U_{CD} = nc_v (T_D - T_C)$

4) $r = \frac{|W_{cycle}|}{Q_{absorbée}} = \frac{|W_{DA} + W_{BC}|}{Q_{AB}}$

$r = \frac{\cancel{nc_v}(T_A - T_D) + \cancel{nc_v}(T_C - T_B)}{\cancel{nc_v}(T_B - T_A)} = \frac{(T_A - T_D) + (T_C - T_B)}{(T_B - T_A)}$

5) si $\frac{a}{b} = \frac{c}{d}$ alors $\left(\frac{a}{b}\right) = \left(\frac{a-c}{b-d}\right) = \left(\frac{c}{d}\right)$.

On a : $\frac{T_C}{T_B} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$

$\frac{T_D}{T_A} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$ $\Rightarrow \frac{T_C}{T_B} = \frac{T_D}{T_A}$

$r = \left|\frac{T_A - T_B}{T_B - T_A} + \frac{T_C - T_D}{T_B - T_A}\right| = \left|-1 + \frac{T_C - T_D}{T_B - T_A}\right| = \left|-1 + \frac{T_C}{T_B}\right| = \left|-1 + \left(\frac{V_1}{V_2}\right)^{\gamma - 1}\right|$

$r = \left|-1 + \left(\frac{1}{a}\right)^{\gamma - 1}\right| = \left|\underbrace{-1 + a^{1-\gamma}}_{<0}\right| = 1 - a^{1-\gamma}$

$\frac{a}{b}$ $\frac{c}{d}$ A.N. : $r = 1 - 9^{-0,4} = 1 - 0,4 = 0,6 = 60\%$

$\frac{3}{6} = \frac{50}{100}$

$\frac{1}{2} = \frac{2}{4} = \frac{1-2}{2-4} = \frac{-1}{-2} = \frac{1}{2}$

$\frac{c-a}{d-b} = \frac{a-c}{b-d} = \frac{1}{2}$

$\frac{a}{b} = \frac{c}{d}$

$\frac{a-c}{b-d} = \frac{a}{b}$

$a\cancel{b} - bc = a\cancel{b} - ad$

$bc = ad$

$\frac{a}{b} = \frac{c}{d}$.

Exercice n°2 : 1) $PV^{\gamma} = cste$ (Laplace)

or $PV = nRT$ (G.P.)

$V = \frac{nRT}{P}$

$P \cdot \left(\frac{nRT}{P}\right)^{\gamma} = cste$

$P^{1-\gamma} \cdot T^{\gamma} = \frac{cste}{(nR)^{\gamma}} = cste$

$A \to B$ (adiab)

$P_A^{1-\gamma} \cdot T_A^{\gamma} = P_B^{1-\gamma} \cdot T_B^{\gamma} \Rightarrow \left(\frac{T_A}{T_B}\right)^{\gamma} = \left(\frac{P_B}{P_A}\right)^{1-\gamma} \Rightarrow \frac{T_A}{T_B} = \left(\frac{P_B}{P_A}\right)^{\frac{1-\gamma}{\gamma}}$ $\square$

de même pour $C \to D$ (adiab)

$P_C^{1-\gamma} \cdot T_C^{\gamma} = P_D^{1-\gamma} \cdot T_D^{\gamma} \Rightarrow \left(\frac{T_C}{T_D}\right)^{\gamma} = \left(\frac{P_D}{P_C}\right)^{1-\gamma} \Rightarrow \frac{T_C}{T_D} = \left(\frac{P_D}{P_C}\right)^{\frac{1-\gamma}{\gamma}}$ $\square$

2)

Transf	$W = -\int_{V_i}^{V_f} P\,dV$	$Q = \Delta U - W$	$\Delta U = nc_v \Delta T$ (G.P.)
$A \to B$ adiab	$W = \Delta U$ car $Q = 0$ $W_{AB} = nc_v(T_B - T_A)$	$Q_{AB} = 0$	$\Delta U = nc_v(T_B - T_A)$.
$B \to C$ isobare	$W_{BC} = -P_B(V_C - V_B)$ $= -P_B\left(\frac{nRT_C}{P_C} - \frac{nRT_B}{P_B}\right)$ $= -nR(T_C - T_B)$	$Q_{BC} = \Delta U_{BC} - W_{BC}$ $= nc_v(T_C - T_B) + nR(T_C - T_B)$ $= (nc_v + nR)(T_C - T_B)$ $= nc_p(T_C - T_B)$	$\Delta U_{BC} = nc_v(T_C - T_B)$
$C \to D$ adiab	$W_{CD} = \Delta U = nc_v(T_D - T_C)$	$Q_{CD} = 0$	$\Delta U_{CD} = nc_v(T_D - T_C)$
$D \to A$ isobare	$W_{DA} = -nR(T_A - T_D)$	$Q_{DA} = nc_p(T_A - T_D)$	$\Delta U_{DA} = nc_v(T_A - T_D)$.

3a) $r = \frac{Q_{BC} + Q_{DA}}{Q_{BC}} = 1 + \frac{Q_{DA}}{Q_{BC}} = 1 + \frac{nc_p(T_A - T_D)}{nc_p(T_C - T_B)} = 1 + \frac{T_A - T_D}{T_C - T_B}$

$r = 1 - \frac{T_D - T_A}{T_B - T_C} = 1 - \frac{T_D}{T_C} = 1 - \left(\frac{P_C}{P_D}\right)^{\frac{1-\gamma}{\gamma}} = 1 - a^{\frac{1-\gamma}{\gamma}}$