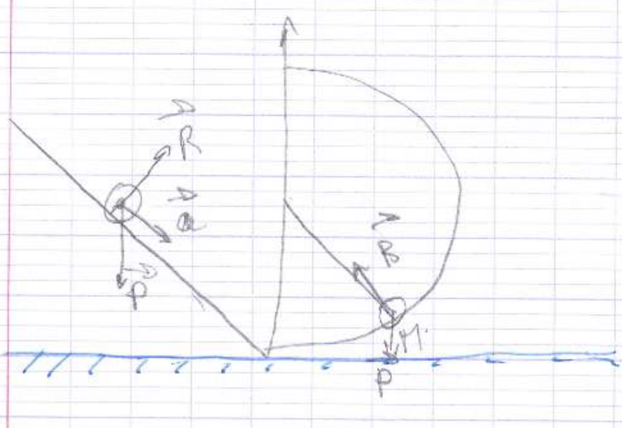
Physique
15/01

Série n° 7 :
Dynamique du point materiel.

Exercice n°1 :



1) Calcul de V_B :

Theorème d'Energie Cinétique :

$$\Delta E_c{}_{A\to B} = \sum W(\vec{F_{ext}})$$

$$\frac{1}{2}mV_B^2 - \underbrace{\frac{1}{2}mV_A^2}_{=0} = W(\vec{P}) + \underbrace{W(\vec{R_N})}_{=0}$$

$$\frac{1}{2}\cancel{m}V_B^2 = \cancel{m}gh$$

$$V_B = \sqrt{gh \times 2}$$

Théorème d'Energie Potentielle de Pesanteur

$$\Delta E_{pp}{}_{A\to B} = \sum W(\vec{F_{ext}}) \qquad \vec{f}_{frott} = \vec{0}$$

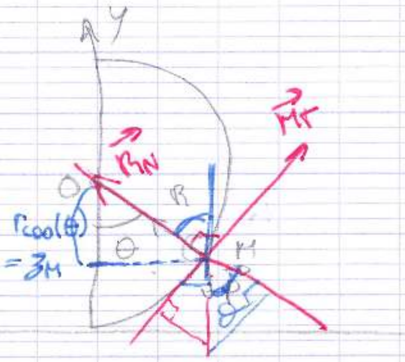
$$E_{m_A} = E_{m_B}$$

$$0 + \cancel{m}g\underbrace{z_A}_{h} = \frac{1}{2}\cancel{m}V_B^2 + \underbrace{mgz_B}_{=0}$$

$$V_B^2 = 2gh$$

$$V_B = \sqrt{2gh}$$

2)



$\vec{f}_{frott} = \vec{0}$

$\vec{P} \begin{cases} P_T = -P\sin(\theta) \\ P_N = -P\cos(\theta) \end{cases}$ $\vec{R} \begin{cases} R_T = 0 \\ R_N = R \end{cases}$

3) Calcul de V_M :

Thm de l'E_c (e) B et M.

$$\frac{1}{2} m V_M^2 - \underbrace{\frac{1}{2} m V_B^2}_{} = \underbrace{W(\vec{P})}_{<0} + \underbrace{W(\vec{R})}_{=0}$$

voir 1)

$$= -mgz = -mgr - mgr\cos(\theta)$$

$$\frac{1}{2} \cancel{m} V_M^2 - \cancel{m}gh \qquad = \cancel{m}gr(1-\cos(\theta))$$

$$V_M^2 = 2g\,[h - r(1-\cos(\theta))].$$

4) Calcul de $\vec{R_N}$:

2e loi de Newton

$$\sum \vec{F_{ext}} = m\vec{a}$$
$$\vec{R} + \vec{P} = m\vec{a}$$

Projection sur $\vec{MN}$:

$$R_N - P\cos(\theta) = ma_N \quad \left(a_N = \frac{V_M^2}{r}\right)$$

$$R_N = mg\cos(\theta) + m\frac{2g}{r}(h - r(1-\cos(\theta)))$$

$$R_N = mg\left[\cos(\theta) + \frac{2h}{r} - 2 + 2\cos(\theta)\right]$$

$$R_N = mg\left[3\cos(\theta) + 2\left(\frac{h}{r} - 1\right)\right]$$

Exercice n°2:

Methodo

1) Calcul V_c :

Utilisation du Théorème d'Energie Mécanique.

$W(\vec{P}) = \vec{0}$ d'où $\Delta E_m = 0$

$\Rightarrow \Delta E_c + \Delta E_{pp} = 0$

$\Rightarrow E_{cc} - E_{cA} + E_{ppc} - E_{ppA} = 0$

$\Rightarrow E_{cc} = E_{cA} + E_{ppA} - E_{ppc}$

$$\frac{1}{2} m V_c^2 = \underbrace{\frac{1}{2} m V_A^2}_{=0} + mgh - mgr.$$

$V_c^2 = 2g(h-r)$ d'où $V_c = \sqrt{2g(h-r)} = \sqrt{10}\ m.s^{-1}$

2a) Calcul de V_M:

$W(\vec{P}) = \vec{0} \Rightarrow E_{m_A} = E_{m_M}$.

$\underbrace{E_{c_A}}_{=0} + \cancel{m}gz_A = E_{c_M} + \cancel{m}gz_M$

$gh = \frac{1}{2} V_M^2 + g(r\sin(\alpha))$.

$V_M^2 = 2g(h - r\sin(\alpha))$

$V_M = \sqrt{2g(h - r\sin(\alpha))}$

A.N:

$V_M = \sqrt{2 \times 10 \times (\underbrace{1 - 0{,}5 \times 0{,}5}_{=0{,}75})}$

$= \sqrt{15}\ m.s^{-1}$

2)b) $a_N = \frac{V_M^2}{r}$

a_N au point M: $= 2g\left(\frac{h}{r} - \sin(\alpha)\right)$.

2c) $\sum \vec{F}_{ext} = m\vec{a}$

$\vec{P} + \vec{R} = m\vec{a}$

$R_N = ma_N - mg\sin(\alpha)$

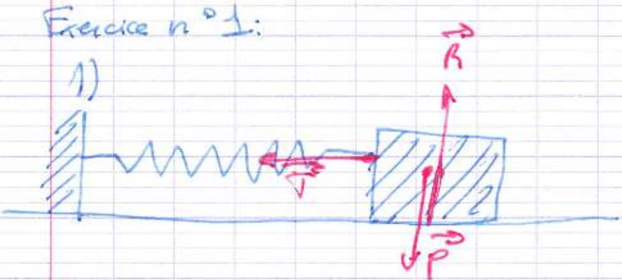
$= m(a_N - g\sin(\alpha))$

$= m(30 - 10 \times \frac{1}{2}) = 25m$.

Serie 8 :

Exercice n° 1 :

1)



Forces ; $\vec{P}, \vec{R}_N, \vec{T}$

2) $\sum \vec{F}_{ext} = m\vec{a} \Rightarrow \vec{P} + \vec{R}_N + \vec{T} = m\vec{a}$ norme $T = k|x|$

Projection sur l'axe Ox.

$0 + 0 - kx = m a_x$

$-kx = m\ddot{x}$ car $\vec{a}\begin{pmatrix} a_x = \ddot{x} \\ a_y = 0 \end{pmatrix}$, $\vec{V}\begin{pmatrix} V_x = \dot{x} \\ V_y = 0 \end{pmatrix}$

$m\ddot{x} + kx = 0$

$\boxed{\ddot{x} + \frac{k}{m} x = 0}$ equation differentielle.

$\equiv \ddot{x} + \omega_0^2 x = 0$ equation type d'un oscillateur harmonique de

pulsation propre

$\omega_0^2 = \frac{k}{m}$

$\omega_0 = \sqrt{\frac{k}{m}}$

3) $x = x_0 \cos(\omega t)$ $\ddot{x} + \frac{k}{m} x = 0$

$\dot{x} = x_0 \omega \sin(\omega t)$

$\ddot{x} = -x_0 \omega^2 \cos(\omega t)$ $-x_0 \omega^2 \cos(\omega t) + \frac{k}{m} x_0 \cos(\omega t)$ $\forall t$

$\Leftrightarrow \underbrace{x_0 \cos(\omega t)}_{x(t)} \left[-\omega^2 + \frac{k}{m} \right] = 0 \quad \forall t$

$\omega^2 + \frac{k}{m} = 0 \Rightarrow \omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$.

$T = \frac{2\pi}{\omega}$ $\begin{cases} \text{periode en sec} \\ \omega = \text{pulsation en rad/s}. \end{cases}$