

## Electrostatique

Exercice 1.

1 Voir sur le schéma

$$\vec{F}_e(M) = \vec{F}_{AM} + \vec{F}_{BM}$$

Projection dans le repère  $(\vec{M}_x, \vec{M}_y)$ :

$$\vec{F}_e = \begin{pmatrix} F_{ex} = F_{Ax} + F_{Bx} \\ F_{ey} = F_{Ay} + F_{By} \end{pmatrix}$$

$$\vec{F}_{AM} = \begin{pmatrix} F_A \cos(\alpha) \\ -F_A \sin(\alpha) \end{pmatrix}$$

$$\vec{F}_{BM} = \begin{pmatrix} F_B \cos(\alpha) \\ F_B \sin(\alpha) \end{pmatrix}$$

Donc  $\vec{F}_e = \begin{pmatrix} 2F_A \cos(\alpha) \\ 0 \end{pmatrix}$  car  $F_A = F_B$ .

$$F_e = \sqrt{F_{ex}^2 + F_{ey}^2} = |F_{ex}| = F_{ex} (> 0)$$

$$\begin{aligned} 2 F_e &= 2 F_A \cos(\alpha) \\ &= 2 \frac{k q Q}{(AM)^2} \times \frac{OM}{AM} \\ &= \frac{2 k q Q}{(a^2 + z^2)} > \frac{2}{\sqrt{a^2 + z^2}} \end{aligned}$$

$$F_{ex}(z) = \frac{2 k q Q |z|}{(a^2 + z^2)^{3/2}} \quad (\text{valable pour tout } M \text{ sur l'axe } (\vec{Ox}))$$

A.N.:  $k = 9 \times 10^9 \text{ SI}$

$$q = 2 \mu\text{C} = 2 \times 10^{-6} \text{ C}$$

$$Q = 4 \mu\text{C} = 4 \times 10^{-6} \text{ C}$$

$$a = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}$$

$$F_{ex}(z) = \frac{2 \times 9 \times 10^9 \times 2 \times 10^{-6} \times 4 \times 10^{-6} z}{(9 \times 10^{-4} + z^2)^{3/2}}$$

$$= \frac{144 \times 10^{-3} z}{(9 \times 10^{-4} + z^2)^{3/2}}$$

## Exercice 2

$$1. E_A(B) = \frac{kq}{a^2}$$

$$E_C(B) = 3 \frac{kq}{a^2}$$

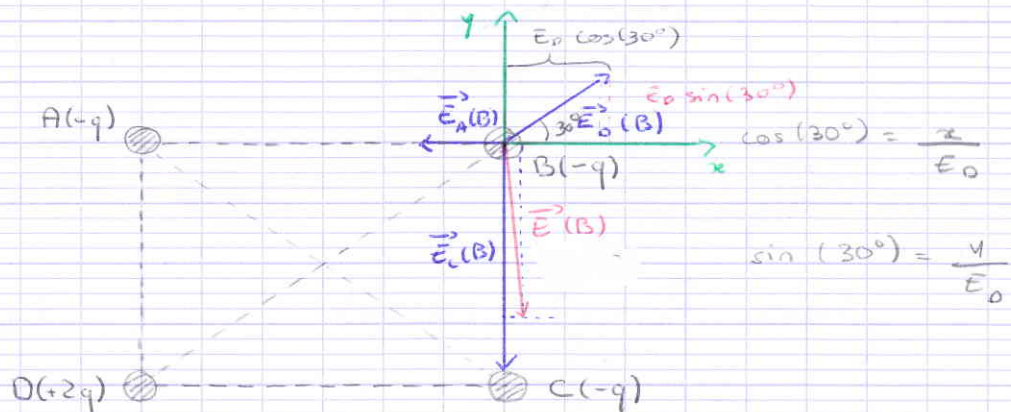
$$E_D(B) = \frac{3}{2} \frac{kq}{a^2}$$

$b^2$  en fonction de  $a^2$

$$\Rightarrow b^2 = \tan(30^\circ)^2 \times a^2$$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 \times a^2$$

$$= \frac{a^2}{3} \Rightarrow b = \frac{a}{\sqrt{3}}$$



$$2. E_A = \begin{pmatrix} -\frac{kq}{a^2} \\ 0 \end{pmatrix} \quad E_C = \begin{pmatrix} 0 \\ -3 \frac{kq}{a^2} \end{pmatrix}$$

$$E_D = \begin{pmatrix} \cos(30^\circ) \times \frac{3}{2} \frac{kq}{a^2} \\ \sin(30^\circ) \times \frac{3}{2} \frac{kq}{a^2} \end{pmatrix}$$

$$\vec{E}_B(B) = \vec{E}_A + \vec{E}_C + \vec{E}_D$$

$$\vec{E}(B) = \begin{pmatrix} E_x(B) = -\frac{kq}{a^2} + \frac{3\sqrt{3}}{4} \frac{kq}{a^2} \\ E_y(B) = -3 \frac{kq}{a^2} + \frac{3}{4} \frac{kq}{a^2} \end{pmatrix}$$

$$= \frac{kq}{4a^2} \begin{pmatrix} -4 + 3\sqrt{3} \\ -12 + 3 \end{pmatrix}$$

$$= \frac{kq}{4a^2} \begin{pmatrix} -4 + 3\sqrt{3} \\ -9 \end{pmatrix} \quad \times$$

Norme:

$$E(B) = \sqrt{E_x(B)^2 + E_y(B)^2} = \frac{kq}{4a^2} \sqrt{(-4 + 3\sqrt{3})^2 + 81}$$

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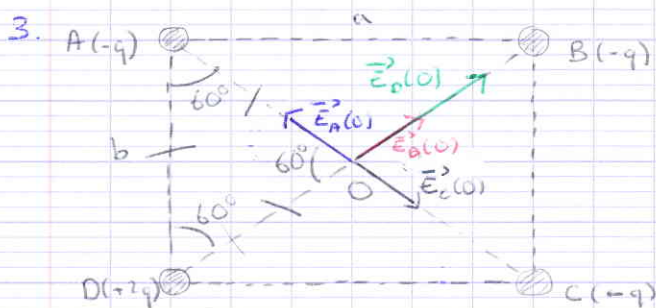
$$E(B) = \frac{kq}{4a^2} \sqrt{124 - 24\sqrt{3}} \approx \frac{kq}{a^2} \times 2,27$$

Pour la représentation :

$$\vec{E}(B) = \frac{kq}{a^2} \begin{pmatrix} -1 + \frac{3\sqrt{3}}{4} \\ -\frac{3}{4} \end{pmatrix} \text{ d'après } *$$

$$\approx \frac{kq}{a^2} \begin{pmatrix} 0,3 \\ -2,25 \end{pmatrix}$$

Voir sur le schéma précédent.



$$\begin{aligned} \vec{E}(O) &= \underbrace{\vec{E}_A(O) + \vec{E}_C(O)}_{=0} + \underbrace{\vec{E}_B(O) + \vec{E}_D(O)}_{\vec{E}_B(O) + 2\vec{E}_B(O)} \\ &= 3\vec{E}_B(O) \end{aligned}$$

Norme :

$$E(O) = 3 \frac{k|q_B|}{OB^2} = \frac{3kq}{b^2} = 3 \frac{kq}{\frac{a^2}{3}} = 9 \frac{kq}{a^2}$$

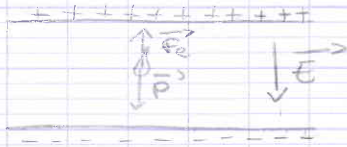
4.  $V(O) = k \sum \frac{q_i}{r_i}$

$$= k \frac{q_A}{OA} + k \frac{q_B}{OB} + k \frac{q_C}{OC} + k \frac{q_D}{OD}$$

$$= \frac{k}{\frac{a}{\sqrt{3}}} (-q - q - q + 2q)$$

$$= -\frac{kq\sqrt{3}}{a}$$

### Exercice 3



$$1. \vec{F}_e = Q\vec{E}$$

si  $Q < 0 \Rightarrow \vec{E}$  opposé à  $\vec{F}_e$

Or  $\vec{E}$  orienté de  $\oplus$  vers  $\ominus \Rightarrow$  plaque sup  $\oplus$   
 plaque inf  $\ominus$

$$2. \text{équilibre} \Leftrightarrow F_e = P \text{ (on norme, } \vec{P} = -\vec{F}_e)$$

$$\Rightarrow |Q|E = mg$$

$$|Q| = \frac{mg}{E}$$

$$= \frac{\rho \times V \times g}{d}$$

$$= \frac{\rho \times \frac{4}{3}\pi R^3 \times g \times d}{d}$$

$$= \frac{\rho \times 4\pi \frac{D^3}{8} \times d \times g}{3d}$$

$$= \frac{\rho \times \pi \times D^3 \times d \times g}{6d}$$

$$V_{\text{cyl}} = \frac{4}{3}\pi R^3, R = \frac{D}{2}$$

$$\text{A.N. } |Q| = \frac{900 \times \pi \times (4 \times 10^{-6})^3 \times 0,15 \times 10}{6 \times 3}$$

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