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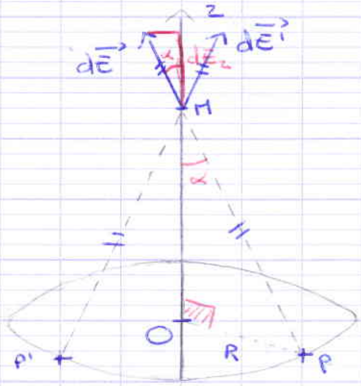
Electrostatique : Distributions de charges continues

Exercice 1

1.



$\vec{dE}$ total porté par $\vec{Oz}$

$\vec{dE}_{tot} = \vec{dE} + \vec{dE'}$

$\Rightarrow \vec{E}$ sera aussi porté par $\vec{Oz}$

$$\vec{E} = \begin{pmatrix} 0 \\ 0 \\ E_z = \int dE_z \end{pmatrix}$$

$$d\ell = R\,d\theta$$

où dE_z = projection du vecteur $\vec{dE}$ sur $\vec{Oz}$

$$= dE \cos(\alpha)$$

$$= \frac{k\,dQ}{(PM)^2} \cos(\alpha)$$

$$= \frac{k\lambda \overbrace{d\ell}^{R d\theta}}{z^2+R^2} \times \frac{OM}{PM}$$

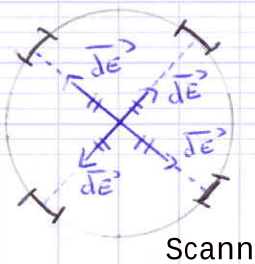
$$= \frac{k\lambda R\,d\theta}{(z^2+R^2)} \times \frac{z}{\sqrt{z^2+R^2}} \quad (*)$$

M étant fixé $\Rightarrow z = cste$ $(R, \lambda, k = cste)$

$$\Rightarrow E_z = \int_0^{2\pi} \underbrace{\left(\frac{k\lambda R z}{(z^2+R^2)^{3/2}} \right)}_{= Cste} d\theta$$

$$= \frac{k\lambda R z}{(z^2+R^2)^{3/2}} \times \int_0^{2\pi} d\theta$$

$$E_z = \frac{2\pi k \lambda R z}{(z^2+R^2)^{3/2}} \quad (*)$$



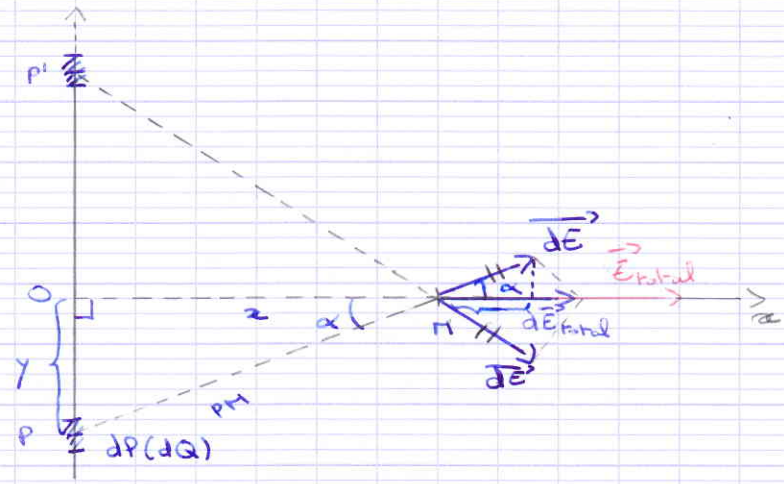
$$\vec{dE}(O) = \frac{k\,\overbrace{dQ}^{\lambda R d\theta}}{\underbrace{(PO)^2}_{R^2}} \times \frac{\vec{PO}}{\underbrace{PO}_{R}} = \frac{k\lambda\,d\theta}{R^2} \vec{PO}$$

$$\vec{E}(O) = \vec{0}$$

2. $dV = \frac{k\,dQ}{PM} = \frac{k \lambda R\,d\theta}{\sqrt{z^2+R^2}}$

$\Rightarrow V(M) = \frac{k \lambda R\, 2\pi}{\sqrt{z^2+R^2}}$

Exercice 2



$\vec{E} = \begin{pmatrix} E_x = \int dE_x \\ E_y = 0 \\ E_z = 0 \end{pmatrix}$

$dE_x = dE \cos(\alpha)$

$= \frac{k\,dQ}{(PM)^2} \times \cos(\alpha)$

$= \frac{k \lambda\,dP}{(PM)^2} \times \cos(\alpha)$

$= \frac{k \lambda\,dy}{(PM)^2} \times \cos(\alpha)$

$\begin{cases} (PM)^2 = f(\alpha, x) \ ? \\ dy = f(x, d\alpha) \ ? \end{cases}$

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* $\underline{(PM)^2 = ?}$

$\cos(\alpha) = \frac{x}{PM} \Rightarrow PM = \frac{x}{\cos(\alpha)} \Rightarrow \boxed{PM^2 = \frac{x^2}{\cos^2(\alpha)}}$ ①

* $dy = ?$

$y = \tan(\alpha) \times x$

$y'(\alpha) = x\,(\tan(\alpha))'$

$\frac{dy}{d\alpha} = x \times \frac{1}{\cos^2(\alpha)} \Rightarrow \boxed{dy = \frac{x}{\cos^2(\alpha)}\, d\alpha}$ ②

① et ② dans dE_x :

$$dE_x = k\lambda \times \frac{\cancel{x}}{\cancel{\cos^2(\alpha)}}\, d\alpha \times \frac{\cancel{\cos^2(\alpha)}}{x^{\cancel{2}}} \times \cos(\alpha)$$

$$\boxed{dE_x = \frac{k\lambda}{x} \cos(\alpha)\, d\alpha}$$

1. $E_x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{k\lambda}{x} \cos(\alpha)\, d\alpha = \frac{k\lambda}{x} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\alpha)\, d\alpha$

$$= \frac{k\lambda}{x} \left[\sin(\alpha)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{k\lambda}{x} \left(\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{-\pi}{2}\right)\right)$$

$$\underline{E(M) = E_x(x) = \frac{2k\lambda}{x}}$$

fil infini

2. Fil AB : $\begin{cases} x = OH = \frac{b}{2} \\ -\frac{\pi}{3} \leq \alpha \leq \frac{\pi}{3} \end{cases} \Rightarrow E_{AB}(O) = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{k\lambda}{x} \cos(\alpha)\, d\alpha$

$$= \frac{k\lambda}{x} \underbrace{\left[\sin(\alpha)\right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}}_{= 2\sin\left(\frac{\pi}{3}\right)}$$

$$= \frac{k\lambda}{\frac{b}{2}} \times \cancel{2} \frac{\sqrt{3}}{\cancel{2}}$$

$$= 2 \frac{k\lambda}{b} \sqrt{3}$$

$$(\text{Or } \tan(30°) = \frac{b}{a} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} \Rightarrow b = \frac{a}{\sqrt{3}}$$

$$E_{AB}(O) = \frac{2k\lambda\sqrt{3}}{\frac{a}{\sqrt{3}}} = 6\frac{k\lambda}{a}$$

Fil BC : $\begin{cases} x = OH' = \frac{a}{2} \\ -\frac{\pi}{6} \leq \alpha \leq \frac{\pi}{6} \end{cases}$

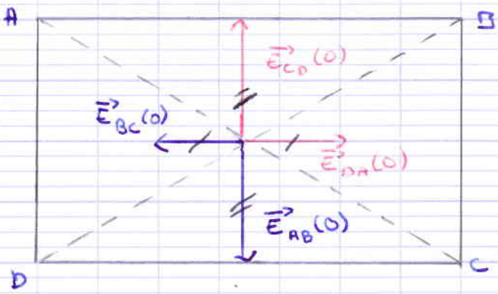
$$E_{BC}(O) = \frac{k\lambda}{\frac{a}{2}} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos(\alpha)\, d\alpha$$

$$= \frac{2k\lambda}{a} \times 2\sin\left(\frac{\pi}{6}\right)$$

$$= \frac{2k\lambda}{a} \times \cancel{2} \times \frac{1}{\cancel{2}}$$

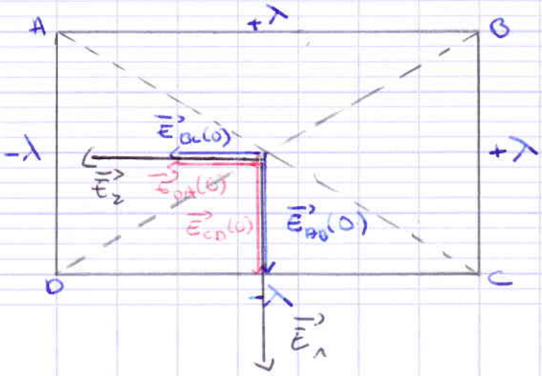
$$= \frac{2k\lambda}{a}$$

b.



$$\vec{E}(O) = \cancel{\vec{E}_{AB}(O)} + \cancel{\vec{E}_{BC}(O)} + \cancel{\vec{E}_{CD}(O)} + \cancel{\vec{E}_{DA}(O)}$$
$$= \vec{0}$$

3.



$\vec{E}_{DA}$: convergent vers (DA) car $(-\lambda)$

$\vec{E}_{CD}$: " " (CD) " "

$\vec{E}(O) = \vec{E}_1(O) + \vec{E}_2(O)$

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$\vec{E}(O) = 2\vec{E}_{AB}(O) + 2\vec{E}_{BC}$

$E_1 = 2E_{AB} = 12\frac{k\lambda}{a}$

$E_2 = 2E_{BC} = 4\frac{k\lambda}{a}$

$E(O) = \sqrt{E_1^2 + E_2^2}$ car $\vec{E_1} \perp \vec{E_2}$

$= \sqrt{144\left(\frac{k\lambda}{a}\right)^2 + 16\left(\frac{k\lambda}{a}\right)^2}$

$= \frac{k\lambda}{a}\sqrt{144+16}$

$= \frac{k\lambda}{a}\sqrt{160}$

$= \frac{k\lambda}{a}\sqrt{16\times 10}$

$= 4\sqrt{10}\,\frac{k\lambda}{a}$