TD Test 4

Name :   **First name :**   **Class :**

Question from the class

Let $\sum f_n$ be a series of functions converging pointwise on an interval I of $\mathbb{R}$.

1. Give the definition of the sequence of the remainder functions (R_n).

2. Give a necessary and sufficient condition for $\sum f_n$ to converge uniformly on I.
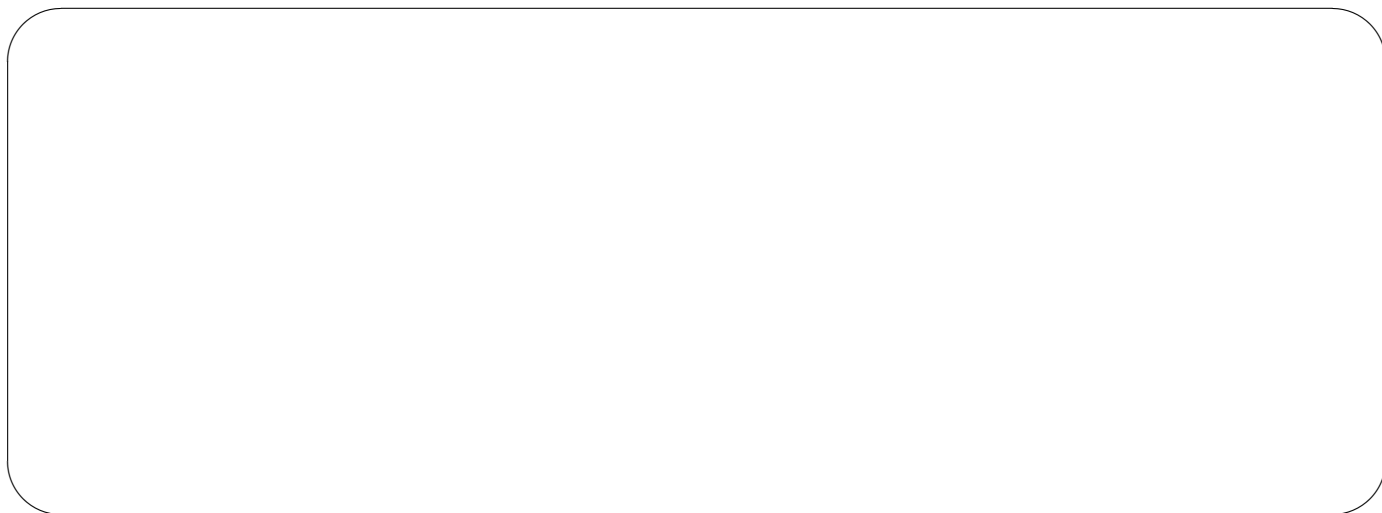
3. Give the definition of the normal convergence of $\sum f_n$ on I.

Exercise 1

Let (f_n) be the sequence of functions defined for every $x \in [0,1]$ by $f_n(x) = \dfrac{nx}{nx+1}$.

1. Study the pointwise and uniform convergence of (f_n) on $[0,1]$.

2. Let $a \in \,]0, 1[$. Study the uniform convergence of (f_n) on $[a, 1]$.

Exercise 2

Let (f_n) be the sequence of functions defined for every $x \in [0, 1]$ by $f_n(x) = \dfrac{ne^{-x} + x^2}{n + x}\cdot$ Study the pointwise and uniform convergence of (f_n) on $[0, 1]$.