$$\vec{F} = \begin{pmatrix} F_x(x,y) = x^2 + 3y \\ F_y(z,y) = z^3 - 2y \\ F_z(x) = 4x \end{pmatrix} \quad F_x \Rightarrow \frac{\partial F_x}{\partial y} = 0 \quad F_y = \frac{\partial F_z}{\partial y} = 0$$

les 1), 2), 5) sont impossibles. On peut calculer : 3) $div(\vec{F})$; 4) $grad(div(\vec{F}))$ et 6) $\vec{rot}(\vec{rot}(\vec{F}))$

$$div(\vec{F}) = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 2x + (-2) + 0 = 2x - 2.$$

$$\vec{grad}(div(\vec{F})) = \vec{grad}(2x-2) = \frac{\partial}{\partial x}(2x-2)\vec{u_x} + \frac{\partial}{\partial y}(2x-2)\vec{u_y} + \frac{\partial}{\partial z}(2x-2)\vec{u_z} = 2u_x + 0 + 0$$

$$\vec{rot}(\vec{rot}(\vec{F})) = \vec{\nabla} \wedge (\vec{\nabla} \wedge \vec{F}).$$

$$\vec{\nabla} \wedge \vec{F} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \wedge \begin{pmatrix} x^2 + 3y \\ z^3 - 2y \\ 4x \end{pmatrix} = \begin{pmatrix} 0 - 3z^2 \\ 0 - 4 \\ 0 - 3 \end{pmatrix}$$

$$\vec{rot}(\vec{rot}(\vec{F})) = - \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \wedge \begin{pmatrix} 3z^2 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 0-0 \\ 6z - 0 \\ 0-0 \end{pmatrix} = \begin{pmatrix} 0 \\ 6z \\ 0 \end{pmatrix}$$

f est une DTE (Différentielle TotaleExacte) ssi ~~f'~~ $\Rightarrow \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

$$\vec{rot}(\vec{grad}(f)) = \vec{\nabla} \wedge (\vec{\nabla} f) = \vec{0}$$

$$div(\vec{rot}(\vec{v})) = 0$$

$$\vec{rot}(\vec{grad}(f)) = \vec{\nabla} \wedge \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \wedge \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}$$

de l'exo 2

Exercice n°2.

2) $div(f\vec{V}) = f\,div(\vec{V}) + \vec{grad}(f).\vec{V} =$

$$div(f\vec{V}) = \frac{\partial}{\partial x}(f.V_x) + \frac{\partial}{\partial y}(f.V_y) + \frac{\partial}{\partial z}(f.V_z)$$

$(uv)'$

$$= \underbrace{\frac{\partial f}{\partial x} v_x}_{u'v} + \underbrace{f \frac{\partial V_x}{\partial x}}_{uv'} + \frac{\partial f}{\partial y} v_y + f \frac{\partial V_y}{\partial y} + \frac{\partial f}{\partial z} v_z + f \frac{\partial V_z}{\partial z}$$

où $f\vec{V} = \begin{array}{|ll|l} f(x,y,z) & V_x(x,y,z) & U_x \\ f(\quad) & V_y(x, \quad) & U_y \\ f(\quad) & V_z(\quad) & U_z \end{array}$

$\Delta f = div(\vec{grad}(f)) = \vec{\nabla} . \vec{\nabla}$?? (Plus de tableau)

$$\Delta\vec{U} = \begin{pmatrix} \Delta U_x \\ \Delta U_y \\ \Delta U_z \end{pmatrix} = \begin{pmatrix} div(grad(U_x)) \\ div(grad(U_y)) \\ div(grad(U_z)) \end{pmatrix} = \begin{pmatrix} \frac{\partial^2}{\partial x^2} U_x + \frac{\partial^2}{\partial y^2} U_x + \frac{\partial^2}{\partial z^2} U_x \\ \frac{\partial^2}{\partial x^2} U_y + \frac{\partial^2}{\partial y^2} U_y + \frac{\partial^2}{\partial z^2} U_y \\ \frac{\partial^2}{\partial x^2} U_z + \frac{\partial^2}{\partial y^2} U_z + \frac{\partial^2}{\partial z^2} U_z \end{pmatrix}$$

Exercice n°1 (suite)

$\Delta\vec{U} = \overrightarrow{grad}(div(\vec{U})) - \overrightarrow{rot}(\overrightarrow{rot}(\vec{U}))$

par déf : $\Delta\vec{F} = \underbrace{(\Delta F_x)}_{f}\vec{U_x} + \underbrace{(\Delta F_y)}_{g}\vec{U_y} + \underbrace{(\Delta F_z)}_{h}\vec{U_z} = \left(\frac{\partial^2}{\partial x^2}(F_x) + \frac{\partial^2}{\partial y^2}(F_x) + \underbrace{\frac{\partial^2}{\partial z^2}(F_x)}_{=0}\right)\vec{U_x}$

$+ \left(\underbrace{\frac{\partial^2 F_y}{\partial x^2}}_{=0} + \underbrace{\frac{\partial^2 F_y}{\partial y^2}}_{=0} + \frac{\partial^2 F_y}{\partial z^2}\right)\vec{U_y} + \left(\underbrace{\frac{\partial^2 F_z}{\partial x^2}}_{=0} + \underbrace{\frac{\partial^2 F_z}{\partial y^2}}_{=0} + \underbrace{\frac{\partial^2 F_z}{\partial z^2}}_{=0}\right)\vec{U_z} = (2+0+0)\vec{U_x} + (6z)\vec{U_y} + (0)\vec{U_z}$.

$\overrightarrow{grad}(div(\vec{F})) = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

$\overrightarrow{rot}(\overrightarrow{rot}(\vec{F})) = \begin{pmatrix} 0 \\ -6z \\ 0 \end{pmatrix}$

on a bien $\overrightarrow{grad}(div(\vec{F})) - \overrightarrow{rot}(\overrightarrow{rot}(\vec{F})) = \begin{pmatrix} 2 \\ 6z \\ 0 \end{pmatrix} = \Delta\vec{F}$.

Exercice n°3 :

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ex 1 (suite) $\Delta(\vec{F})$?

vérifier l'identité remarquable : $\Delta\vec{U} = \vec{grad}(div(\vec{u})) - \vec{rot}(\vec{rot}(\vec{u}))$

$$\Delta\vec{F} = \begin{pmatrix} \Delta F_x \\ \Delta F_y \\ \Delta F_z \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 F_x}{\partial x} + \frac{\partial^2 F_x}{\partial y} + \frac{\partial^2 F_x}{\partial z} \\ \frac{\partial^2 F_y}{\partial x} + \frac{\partial^2 F_y}{\partial y} + \frac{\partial^2 F_z}{\partial z} \\ \frac{\partial^2 F_z}{\partial x} + \frac{\partial^2 F_z}{\partial y} + \frac{\partial^2 F_z}{\partial z} \end{pmatrix} = \begin{pmatrix} 2 \\ 6z \\ 0 \end{pmatrix}$$

$$\vec{grad}(div(\vec{F})) = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \quad \text{et} \quad \vec{rot}(\vec{rot}(\vec{F})) = \begin{pmatrix} 0 \\ 6z \\ 0 \end{pmatrix}$$

On a bien $\Delta\vec{F} = \vec{grad}(div(\vec{F})) - \vec{rot}(\vec{rot}(\vec{F}))$

ex 2 (suite) 3.
$$\begin{aligned}\Delta(fg) &= div(\vec{grad}(fg)) = div(f\,\vec{grad}(g) + g\,\vec{grad}(f)) \\ &= div(f\,\vec{grad}(g)) + div(g\,\vec{grad}(f)) \\ &= f.div(\vec{grad}(g)) + \vec{grad}(g).\vec{grad}(f) \\ &\quad + g.div(\vec{grad}(f)) + \vec{grad}(f).\vec{grad}(g) \\ &= f.\Delta g + 2\,\vec{grad}(g).\vec{grad}(f) + g\Delta f\end{aligned}$$

2e méthode

$$\Delta(fg) = div(\vec{grad}(fg))$$

$$= \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial x}(fg) \\ \frac{\partial}{\partial y}(fg) \\ \frac{\partial}{\partial z}(fg) \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \begin{pmatrix} g\frac{\partial f}{\partial x} + f\frac{\partial g}{\partial x} \\ g\frac{\partial f}{\partial y} + f\frac{\partial g}{\partial y} \\ g\frac{\partial f}{\partial z} + f\frac{\partial g}{\partial z} \end{pmatrix}$$

$$= \frac{\partial(g\partial f)}{(\partial x)^2} + \frac{\partial(f\partial g)}{(\partial x)^2} + \frac{\partial(g\partial f)}{(\partial y)^2} + \frac{\partial(f\partial g)}{(\partial y)^2} + \frac{\partial(g\partial f)}{(\partial z)^2} + \frac{\partial(\ \partial\ )}{(\partial z)^2}$$

$$= \frac{\partial g \partial f}{(\partial x)^2} + g\frac{\partial^2 f}{(\partial x)^2} + \frac{\partial f \partial g}{(\partial x)^2} + f\frac{\partial^2 g}{(\partial x)^2} + \dots$$

$$= f.\Delta g + 2\,\vec{grad}(f).\vec{grad}(g) + g.\Delta f$$

ex3.

$$\vec{E} = \begin{pmatrix} 0 \\ 0 \\ E_z(x,t) \end{pmatrix}$$

1.
$$\Delta\vec{E} = \begin{pmatrix} \Delta E_x = 0 \\ \Delta E_y = 0 \\ \Delta E_z = -E_0 k^2 \cos(kx-\omega t) \end{pmatrix}$$ on ne dérive que par rapport à x, y, z

$$= (-E_0 k^2 \cos(kx-\omega t))\vec{e_z}$$
$$= -k^2\vec{E}$$

$$\frac{\partial^2\vec{E}}{\partial t^2} = \begin{pmatrix} 0 \\ 0 \\ -E_0\, \omega^2 \cos(kx-\omega t) \end{pmatrix} = -\omega^2\vec{E}$$

2. $\Delta\vec{E} - \mu_0 \varepsilon_0 \dfrac{\partial^2 \vec{E}}{\partial t^2} = \vec{0}$

→ équat° de propagat° du chp $\vec{E}$ obtenu à l'aide des 4 équat° de Maxwell et l'identité remarquable

a. $-k^2\vec{E} + \omega^2\vec{E}\mu_0\varepsilon_0 = \vec{0}$
$\Rightarrow \vec{E}(-k^2 + \mu_0\varepsilon_0\omega^2) = \vec{0}$
$\vec{E}$ ne peut pas être nulle dc $-k^2 + \mu_0\varepsilon_0\omega^2 = 0$
$\iff \dfrac{\omega}{k} = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$

b. $\Delta\vec{E} - \mu_0\varepsilon_0 \dfrac{\partial^2\vec{E}}{\partial t^2} = \vec{0}$

project° sur (e_z). $\Delta E_z - \mu_0\varepsilon_0 \dfrac{\partial^2 E_z}{\partial t^2} = 0 \iff \mu_0\varepsilon_0 = \dfrac{\Delta E_z}{\frac{\partial^2 E_z}{\partial t^2}}$

dc $[\mu_0\varepsilon_0] = \dfrac{[\Delta][E_z]}{[\frac{\partial^2}{\partial t^2}][E_z]} = \dfrac{m^{-2}}{s^{-2}} = \dfrac{1}{m^2 \cdot s^{-2}} = \dfrac{1}{(m \cdot s^{-1})^2}$

$\mu_0\varepsilon_0$ est homogène à l'inverse du carré d'une vitesse.
dc $\dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$ est homogène à une vitesse.

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On pose donc $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ = célérité des O.E.M (vide ou air)
ondes électromagnétiq

L'équat° de propagat° devient $\Delta \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0}$

Calcul de c.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{4\pi . 10^{-7} . \frac{10^{-9}}{4 \times 9 \pi}}} = \frac{1}{\sqrt{\frac{10^{-16}}{9}}} = 3.10^8 \text{ m/s}$$