Ex 1 TD Complex (1)

1) $\sum_{0\leq i<N} \sum_{j=1}^{N} 1 \iff \sum_{0\leq i<N} N = N^2$.

2) $\sum_{0\leq i<N} \sum_{j=1}^{N/2} 1 \iff \sum_{0\leq i<N} \left\lfloor \frac{N}{2} \right\rfloor = N \cdot \lfloor N/2 \rfloor$

3) $\sum_{0\leq i<N} \sum_{j=0}^{i} 1 \Rightarrow \sum_{0\leq i<N} (i+1) = \frac{(N+1)N}{2}$ $\qquad \frac{(pt+dt)nt}{2}$

4) $\sum_{0\leq i<N} \sum_{j=1}^{N/2} 1 = \sum_{0\leq i<N} \lfloor N/2 \rfloor \Rightarrow N \cdot \lfloor N/2 \rfloor$.

$\sum_{0\leq i<N} \sum_{j=0}^{\lfloor \log_2(N-1) \rfloor} 1 = \sum_{0\leq i<N} \lfloor \log_2(N-1) \rfloor + 1 = N(\lfloor \log_2(N-1) \rfloor + 1)$

$i \leq j < N \iff 1 \leq 2^k \leq N$

$i \leq 2^k \leq N-1 \iff 0 \leq k \leq \log_2(N-1)$

$0 \leq k \leq \lfloor \log_2(N-1) \rfloor$.

5) $\sum_{0\leq i<N} \sum_{i\leq j\leq N-3} 1 = \sum_{0\leq i<N} (N-2-i) = \frac{(N-1)(N-2)}{2} = N$

6) $\sum_{0\leq i<N} \sum_{0\leq k\leq \lfloor i/2 \rfloor} 1 = \sum_{0\leq i<N} \lfloor i/2 \rfloor + 1$

N paire:

$$\frac{2(1+(N/2)-1+1)(N/2)}{2} =$$

$$2\frac{((N/2)+1)N/2}{2} = \frac{(N+2)N}{4}$$

N impair:

$A(N) = A(N-1) + \left\lfloor \frac{N-1}{2} \right\rfloor + 1$.

$A(N) = \frac{(N-1)(N-1+2)}{4} + \frac{N-1}{2} + 1$

$A(N) = \frac{(N-1)(N+1)}{4} + \frac{N-1}{2} + 1$.

$A(N) = \frac{(N+1)^2}{4}$

Exercice n° 2 :

$$\left(\sum_{y=0}^{n-1} \sum_{x=n-y}^{n+y} 1\right) + \left(\sum_{y=1}^{n-1} \sum_{x=1}^{2n-1} 1\right) = \sum_{y=0}^{n-1} (2y+1) + \sum_{y=1}^{n-1} (2n-1)$$

$$= \frac{(1+2(n-1)+1)n}{2} + (n-1)(2n-1) = n^2 + (n-1)(2n-1) = 3n^2 - 3n + 1.$$

Exercice n°3 :

1) $\frac{(1000+1)1000}{2}$

2) $\log_{10}(42) = 1,b$ $\qquad$ $\log_{10}(1708019,8) = 6,b$

$\log_{10}(4,2) = 0,b$ $\qquad$ $\log_{10}(1,70) = 0,b$

$\log_{10}(4,2^{10}) = a,b$

Exercice n°4 :

1) $n_i = f - 1$.

2) $h+1 \leqslant f \leqslant 2^h$

$h \leqslant n_i \leqslant 2^h - 1$

$2h+1 \leqslant n \leqslant 2^{h+1} - 1$

3) $\lceil \log_2(f) \rceil \leqslant h$

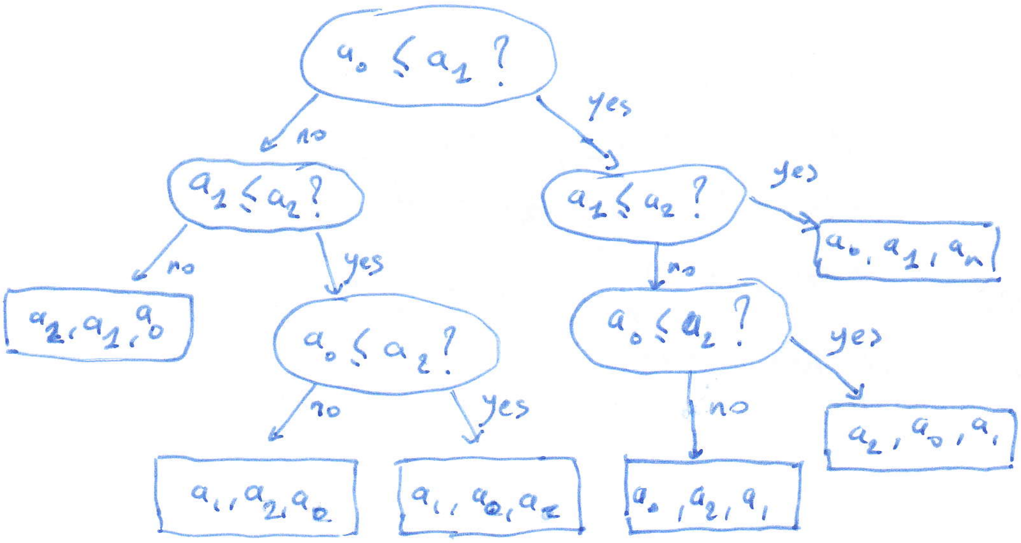
$\lceil \log_2(n_i+1) \rceil \leqslant h$

$\lceil \log_2(n+1) \rceil \leqslant h-1 \Leftrightarrow \lceil \log_2(n+1) \rceil - 1 \leqslant h$

$\Rightarrow \lfloor \log_2(n+1) \rfloor \leqslant h.$

Exercice n°5

a_0, a_1, a_2 :



$\log_2(n!) = \Theta(n.\log(n))$